# Series de Fourier

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## Ejercicios

Para las siguientes funciones periódicas, grafica cada función, estudia si la función es par o impar, halla la serie de Fourier real y suma la(s) series dadas utilizando la serie de Fourier encontrada:

$$f_1(x) = \begin{cases} 1 & 0 < x < \pi \\ -1 & -\pi < x < 0 \end{cases}, \qquad \sum_{n=0}^{\infty} \frac{(-1)^n}{1+2n} \qquad \sum_{n=0}^{\infty} \frac{1}{(1+2n)^2}$$

$$f_2(x) = \begin{cases} x & 0 < x < \pi \\ -x & -\pi < x < 0 \end{cases}, \qquad \sum_{n=0}^{\infty} \frac{1}{(2n+1)^2}, \qquad \sum_{n=1}^{\infty} \frac{1}{(2n+1)^4}$$

$$f_3(x) = x, \qquad -\pi < x < \pi, \qquad \sum_{k=0}^{\infty} \frac{(-1)^{k+1}}{2k+1}, \qquad \sum_{k=1}^{\infty} \frac{1}{k^2}$$

$$f_4(x) = x, \qquad 0 < x < 2\pi \qquad 1 + \frac{1}{3} - \frac{1}{5} - \frac{1}{7} + \frac{1}{9} + \frac{1}{11} - ..$$

Pista: usa que $\frac{1}{2} - \frac{1}{6} + \frac{1}{10} - \frac{1}{14} + ... = \frac{1}{2}\left(1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + ...\right) = \frac{\pi}{4}$

$$f_5(x) = |\operatorname{sen} x|, \qquad -\pi < x < \pi \qquad \sum_{n=1}^{\infty} \frac{1}{4n^2-1}, \qquad \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{4n^2-1}$$

$$f_6(x) = \begin{cases} \sin x & 0 < x < \pi \\ 0 & -\pi < x < 0 \end{cases} \qquad \sum_{n=1}^{\infty} \frac{1}{(4n^2-1)^2}$$

$$f_7(x) = \begin{cases} \cos x & 0 < x < \pi \\ -\cos x & -\pi < x < 0 \end{cases}, \qquad \sum_{k=0}^{\infty} \frac{(2k+1)(-1)^k}{4(2k+1)^2-1}, \qquad \sum_{n=1}^{\infty} \frac{n^2}{(4n^2-1)^2}$$

$$f_8(x) = x^2, \qquad -\pi < x < \pi, \qquad \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2}$$

$$f_9(x) = x(\pi - x), \qquad 0 < x < \pi, \qquad \sum_{n=1}^{\infty} \frac{1}{n^2}, \qquad \sum_{n=0}^{\infty} \frac{1}{2n+1^2}, \qquad \sum_{n=1}^{\infty} \frac{1}{n^4}$$

$$f_{10}(x) = x(\pi - x)(\pi + x), \qquad -\pi < x < \pi, \qquad \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k+1)^3}, \qquad \sum_{n=1}^{\infty} \frac{1}{n^6}$$

$$f_{11}(x) = \begin{cases} 0 & 0 < x < \pi - \alpha \\ 1 & \pi - \alpha < x < \pi + \alpha \\ 0 & \pi + \alpha < x < 2\pi \end{cases}, \qquad \sum_{k=1}^{\infty} \frac{(-1)^k \operatorname{sen} k\alpha}{k}, \qquad \sum_{k=1}^{\infty} \frac{(\operatorname{sen} k\alpha)^2}{k^2}$$

$$f_{12}(x) = \begin{cases} x(\pi - x) & 0 < x < \pi \\ x(\pi - x) & -\pi < x < 0 \end{cases}$$

$$f_{13}(x) = \operatorname{sen} \alpha x, \quad -\pi < x < \pi \qquad \sum_{k=0}^{\infty} \frac{(-1)^k (2k+1)}{(2k+1)^2 - \alpha^2}, \qquad \sum_{k=1}^{\infty} \frac{(k^2}{(k^2 - \alpha^2)^2}$$

$$\alpha \notin \mathbb{Z}$$

$$f_{14}(x) = \sin^4 x, \quad -\pi < x < \pi$$

$$f_{15}(x) = \operatorname{senh} \alpha x, \quad -\pi < x < \pi \qquad \sum_{k=0}^{\infty} \frac{(-1)^k (2k+1)}{(2k+1)^2 + \alpha^2}, \qquad \sum_{k=1}^{\infty} \frac{(k^2}{(k^2 + \alpha^2)^2}$$

$$f_{16}(x) = \arctan\left(\frac{\alpha \operatorname{sen} x}{1 - \alpha \cos x}\right), \qquad \sum_{k=0}^{\infty} \frac{(-1)^k \alpha^{2k+1}}{2k+1}$$

$$-\pi < x < \pi \quad |\alpha| < 1$$

$$f_{17}(x) = \log(1 - 2\alpha \cos x + \alpha^2), \qquad \sum_{k=1}^{\infty} \frac{\alpha^k}{k}$$

$$-\pi < x < \pi \quad |\alpha| < 1$$

$$f_{18}(x) = \log(|\cos x/2|), \quad -\pi < x < \pi, \qquad \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n}, \qquad \int_{-\pi}^{\pi} \log^2(|\cos(x/2)|)dx$$

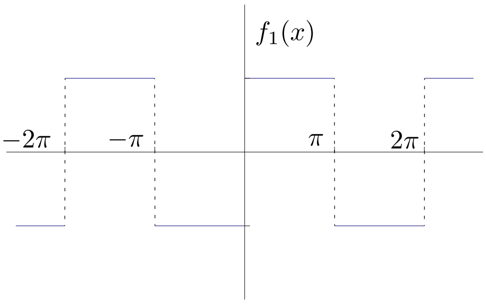
$$f_{19}(x) = \begin{cases} (x+2)^2 & -2 < x < -1 \\ 1 & -1 < x < 1 \\ (x-2)^2 & 1 < x < 2 \end{cases}, \qquad \sum_{n=1}^{\infty} \frac{1}{n^2} \quad , \qquad \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2}, \qquad \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)^3}$$

$$f_{20}(x) = \begin{cases} x^2 & 0 < x < 1 \\ -x^2 + 2x & 1 < x < 2 \end{cases}, \qquad \sum_{n=1}^{\infty} \frac{1}{n^2}, \qquad \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)^3}$$

$$f_{21}(x) = \begin{cases} e^x & 0 < x < 1 \\ e^{-x} & 1 < x < 2 \end{cases}, \qquad \sum_{n=1}^{\infty} \frac{e^{-1} - (-1)^n}{1 + n^2\pi^2}$$

$$f_1(x) = \begin{cases} 1 & 0 < x < \pi \\ -1 & -\pi < x < 0 \end{cases}$$

$$f_1(x) = \frac{4}{\pi} \sum_{n=0}^{\infty} \frac{\operatorname{sen}((2n+1)x)}{2n+1}$$

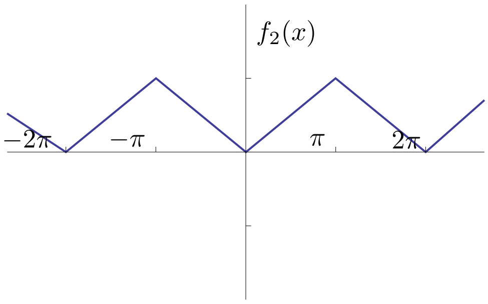



$$x = \frac{\pi}{2} \Rightarrow \sum_{n=0}^{\infty} \frac{(-1)^n}{1+2n} = \frac{\pi}{4}$$

$$\text{Parseval} \Rightarrow \sum_{n=0}^{\infty} \frac{1}{(1+2n)^2} = \frac{\pi^2}{8}$$

$$f_2(x) = \begin{cases} x & 0 < x < \pi \\ -x & -\pi < x < 0 \end{cases}$$

$$f_2(x) = \frac{\pi}{2} - \frac{4}{\pi} \sum_{n=0}^{\infty} \frac{\cos((2n+1)x)}{(2n+1)^2}$$

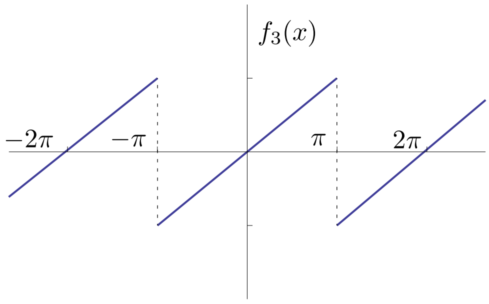



$$x = 0 \Rightarrow \sum_{n=0}^{\infty} \frac{1}{(2n+1)^2} = \frac{\pi^2}{8}$$

$$\text{Parseval} \Rightarrow \sum_{n=0}^{\infty} \frac{1}{(2n+1)^4} = \frac{\pi^4}{96}$$

$$f_3(x) = x, \qquad -\pi < x < \pi$$

$$f_3(x) = 2 \sum_{n=1}^{\infty} \frac{(-1)^{n+1} \operatorname{sen} nx}{n}$$

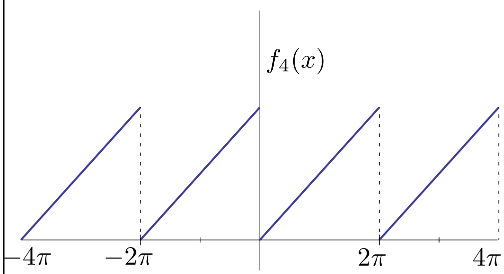



$$x = \frac{\pi}{2} \Rightarrow \sum_{n=0}^{\infty} \frac{(-1)^{n+1}}{(2n+1)^2} = \frac{\pi}{4}$$

$$\text{Parseval} \Rightarrow \sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$$

$$f_4(x) = x, \qquad 0 < x < 2\pi$$

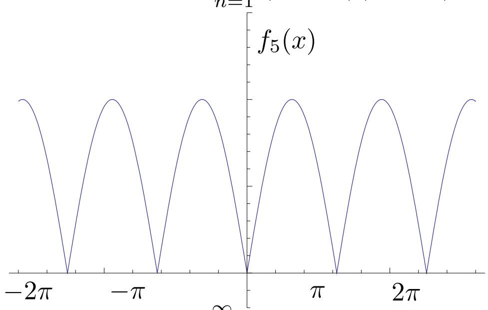
$$f_4(x) = \pi - 2 \sum_{n=1}^{\infty} \frac{\operatorname{sen} nx}{n}$$




$$x = \frac{\pi}{4} \Rightarrow \frac{1}{1} + \frac{1}{3} - \frac{1}{5} - \frac{1}{7} + \frac{1}{9} + \frac{1}{11} - - = \frac{\sqrt{2}\pi}{4}$$

$$f_5(x) = |\operatorname{sen} x|, \qquad -\pi < x < \pi$$

$$f_5(x) = \frac{2}{\pi} - \frac{4}{\pi} \sum_{n=1}^{\infty} \frac{\cos(2nx)}{(2n-1)(2n+1)}$$

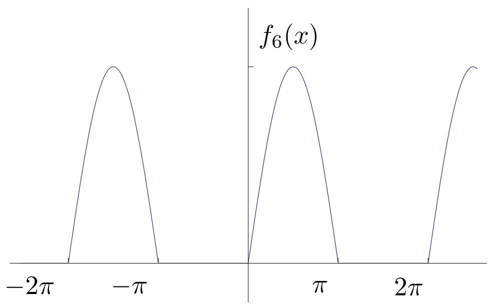



$$x = 0 \Rightarrow \sum_{n=1}^{\infty} \frac{1}{4n^2-1} = \frac{1}{2}$$

$$x = \frac{\pi}{2} \Rightarrow \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{4n^2-1} = \frac{\pi}{4} - \frac{1}{2}$$

$$f_6(x) = \begin{cases} \sin x & 0 < x < \pi \\ 0 & -\pi < x < 0 \end{cases}$$

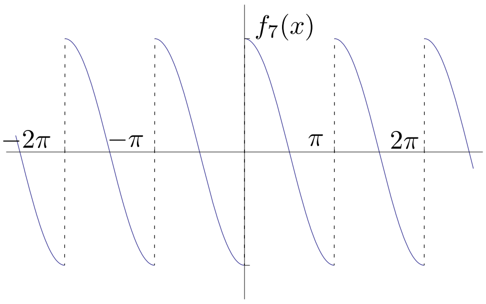
$$f_6(x) = \frac{1}{\pi} + \frac{1}{2} \operatorname{sen} x - \frac{2}{\pi} \sum_{n=1}^{\infty} \frac{\cos(2nx)}{(2n-1)(2n+1)}$$




$$\text{Parseval} \Rightarrow \sum_{n=1}^{\infty} \frac{1}{(4n^2-1)^2} = \frac{\pi^2}{16} - \frac{1}{2}$$

$$f_7(x) = \begin{cases} \cos x & 0 < x < \pi \\ -\cos x & -\pi < x < 0 \end{cases}$$

$$f_7(x) = \frac{8}{\pi} \sum_{n=1}^{\infty} \frac{n \operatorname{sen}(2nx)}{(2n-1)(2n+1)}$$

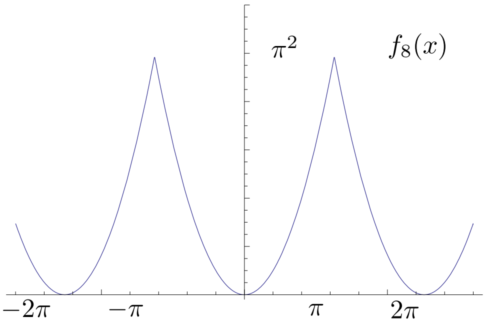



$$x = \frac{\pi}{4} \Rightarrow \sum_{k=0}^{\infty} \frac{(2k+1)(-1)^k}{4(2k+1)^2 - 1} = \frac{\sqrt{2}\pi}{16}$$

$$\text{Parseval} \Rightarrow \sum_{n=1}^{\infty} \frac{n^2}{(4n^2-1)^2} = \frac{\pi^2}{64}$$

$$f_8(x) = x^2, \qquad -\pi < x < \pi$$

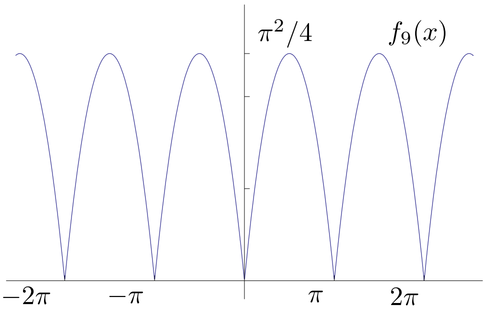
$$f_8(x) = \frac{\pi^2}{3} - 4 \sum_{n=1}^{\infty} \frac{(-1)^n \cos nx}{n^2}$$




$$x = 0 \Rightarrow \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2} = \frac{\pi^2}{12}$$

$$f_9(x) = x(\pi - x), \qquad 0 < x < \pi$$

$$f_9(x) = \frac{\pi^2}{6} - \sum_{n=1}^{\infty} \frac{\cos 2nx}{n^2}$$




$$x = 0 \Rightarrow \sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$$

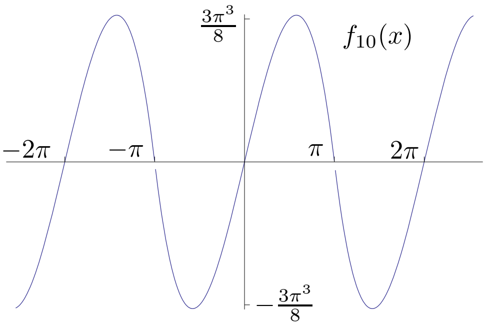
Usando tambien $f_8(x)$

$$\Rightarrow \sum_{n=0}^{\infty} \frac{1}{(2n+1)^2} = \frac{\pi^2}{8}$$

$$\text{Parseval} \Rightarrow \sum_{n=1}^{\infty} \frac{1}{n^4} = \frac{\pi^2}{90}$$

$$f_{10}(x) = x(\pi - x)(\pi + x), \qquad -\pi < x < \pi$$

$$f_{10}(x) = 12 \sum_{n=1}^{\infty} \frac{(-1)^n \operatorname{sen} nx}{n^3}$$

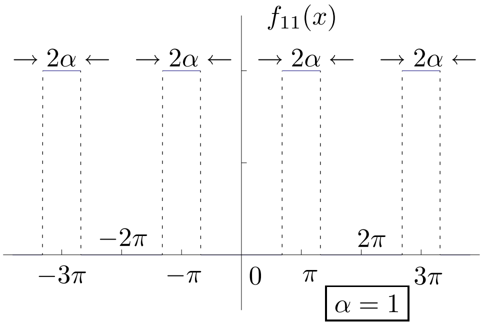



$$x = \frac{\pi}{2} \Rightarrow \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k+1)^3} = \frac{\pi^3}{32}$$

$$\text{Parseval} \Rightarrow \sum_{n=1}^{\infty} \frac{1}{n^6} = \frac{\pi^6}{945}$$

$$f_{11}(x) = \begin{cases} 0 & 0 < x < \pi - \alpha \\ 1 & \pi - \alpha < x < \pi + \alpha \\ 0 & \pi + \alpha < x < 2\pi \end{cases}$$

$$f_{11}(x) = \frac{\alpha}{\pi} + \frac{2}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^n \operatorname{sen} n\alpha \cos nx}{n}$$

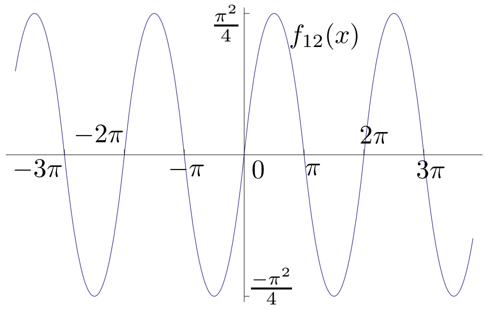



$$x = 0 \Rightarrow \sum_{k=1}^{\infty} \frac{(-1)^k \operatorname{sen} k\alpha}{k} = \frac{-\alpha}{2}$$

$$\text{Parseval} \Rightarrow \sum_{k=1}^{\infty} \frac{(\operatorname{sen} k\alpha)^2}{k^2} = \frac{\alpha}{2}(\pi - \alpha)$$

$$f_{12}(x) = \begin{cases} x(\pi - x) & 0 < x < \pi \\ x(\pi - x) & -\pi < x < 0 \end{cases}$$

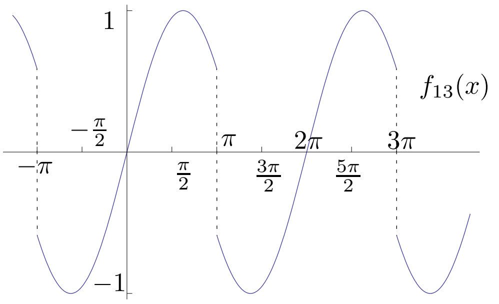
$$f_{12}(x) = \frac{8}{\pi} \sum_{n=0}^{\infty} \frac{\operatorname{sen}((2n+1)x)}{(2n+1)^3}$$




$$x = \frac{\pi}{2} \Rightarrow \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k+1)^3} = \frac{\pi^3}{32}$$

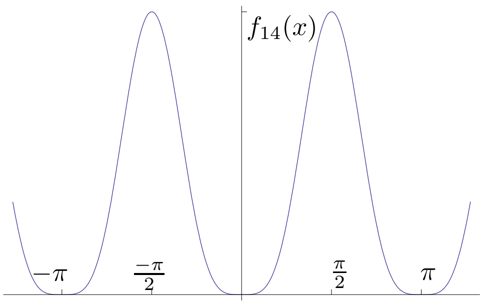
$$\text{Parseval} \Rightarrow \sum_{k=0}^{\infty} \frac{1}{(2k+1)^6} = \frac{\pi^6}{960}$$

$$f_{13}(x) = \operatorname{sen} \alpha x, \quad -\pi < x < \pi$$
$$\alpha \notin \mathbb{Z}$$
$$f_{13}(x) = \frac{2 \operatorname{sen} \alpha\pi}{\pi} \sum_{n=1}^{\infty} \frac{n \operatorname{sen} nx}{n^2 - \alpha^2}$$

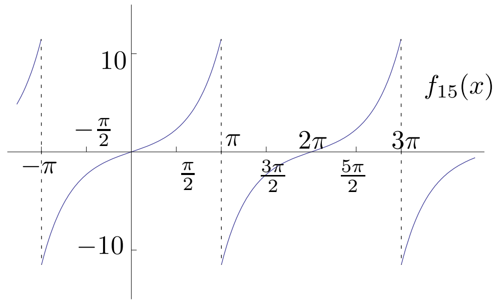



$$x = \frac{\pi}{2} \Rightarrow \sum_{k=0}^{\infty} \frac{(-1)^k(2k+1)}{(2k+1)^2 - \alpha^2} = \frac{\pi}{4\cos(\alpha\pi/2)}$$
$$\text{Parseval} \Rightarrow \sum_{k=1}^{\infty} \frac{k^2}{(k^2-\alpha^2)^2} = \frac{\alpha\pi^2 \csc^2(\alpha\pi) - \pi\cot(\alpha\pi)}{4\alpha}$$

$$f_{14}(x) = \sin^4 x, \quad -\pi < x < \pi$$
$$f_{14}(x) = \frac{3}{4} - \frac{1}{2}\cos 2x + \frac{1}{8}\cos 4x$$

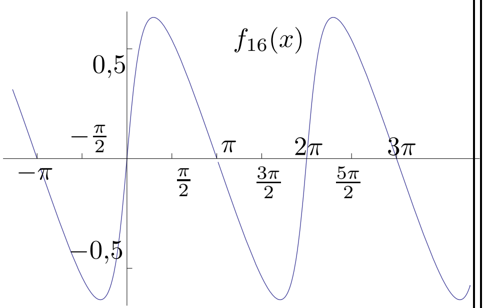



$$f_{15}(x) = \operatorname{senh} \alpha x, \quad -\pi < x < \pi$$
$$f_{15}(x) = \frac{2 \operatorname{senh} \alpha\pi}{\pi} \sum_{n=1}^{\infty} \frac{n(-1)^{n+1} \operatorname{sen} nx}{n^2 + \alpha^2}$$



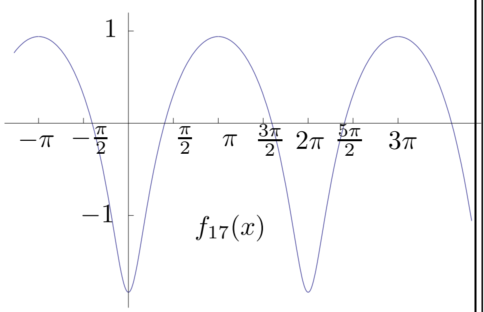

$$x = \frac{\pi}{2} \Rightarrow \sum_{k=0}^{\infty} \frac{(-1)^k(2k+1)}{(2k+1)^2 + \alpha^2} = \frac{\pi}{4\cosh(\alpha\pi/}$$
$$\text{Parseval} \Rightarrow \sum_{k=1}^{\infty} \frac{k^2}{(k^2+\alpha^2)^2} = \frac{\pi\coth(\alpha\pi) - \alpha\pi^2 \operatorname{csch}^2(\alpha\pi)}{4\alpha}$$

$$f_{16}(x) = \arctan\left(\frac{\alpha \operatorname{sen} x}{1 - \alpha\cos x}\right)$$
$$-\pi < x < \pi \quad |\alpha| < 1$$
$$f_{16}(x) = \sum_{n=1}^{\infty} \frac{\alpha^n \operatorname{sen} nx}{n}$$



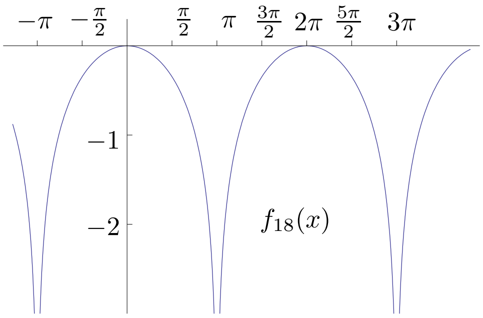

$$x = \frac{\pi}{2} \Rightarrow \sum_{k=0}^{\infty} \frac{(-1)^k \alpha^{2k+1}}{2k+1} = \operatorname{atan} \alpha$$

$$f_{17}(x) = \log(1 - 2\alpha\cos x + \alpha^2)$$
$$-\pi < x < \pi \quad |\alpha| < 1$$
$$f_{17}(x) = -2\sum_{n=1}^{\infty} \frac{\alpha^n \cos nx}{n}$$




$$x = 0 \Rightarrow \sum_{n=1}^{\infty} \frac{\alpha^n}{n} = \log(1+\alpha^2)$$

$$f_{18}(x) = \log(|\cos x/2|), \quad -\pi < x < \pi$$
$$f_{18}(x) = -\log 2 - \sum_{n=1}^{\infty} \frac{(-1)^{n+1}\cos nx}{n}$$

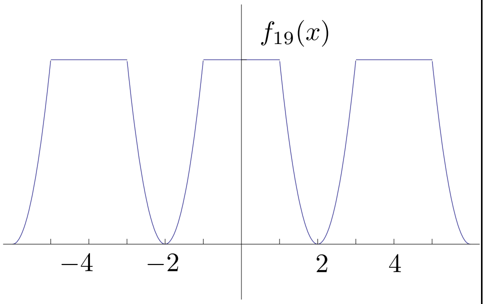



$$x = 0 \Rightarrow \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} = \log 2$$
$$\text{Parseval} \Rightarrow \int_{-\pi}^{\pi} \log^2(|\cos(x/2)|)dx = 2\pi\log^2 2 + \frac{\pi^3}{6}$$

$$f_{19}(x) = \begin{cases} (x+2)^2 & -2 < x < -1 \\ 1 & -1 < x < 1 \\ (x-2)^2 & 1 < x < 2 \end{cases}$$

$$f_{19}(x) = \frac{2}{3} + \frac{2}{\pi^2} \sum_{k=1}^{\infty} \frac{(-1)^k \cos \pi k x}{k^2} +$$

$$\frac{16}{\pi^3} \sum_{k=0}^{\infty} \frac{(-1)^k \cos((2k+1)\pi x/2)}{(2k+1)^3}$$




$$x = 1 \Rightarrow \sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$$

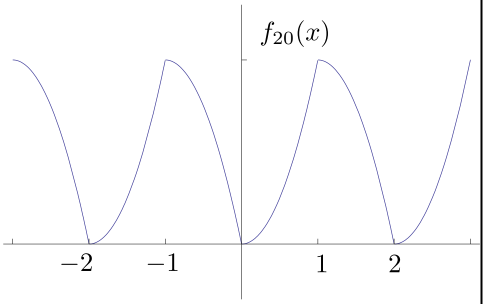
$$x = 0 \Rightarrow \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2} = \frac{\pi^2}{12}$$

$$x = 2 \Rightarrow \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)^3} = \frac{\pi^3}{32}$$

$$f_{20}(x) = \begin{cases} x^2 & 0 < x < 1 \\ -x^2 + 2x & 1 < x < 2 \end{cases}$$

$$f_{20}(x) = \frac{1}{2} - \frac{4}{\pi^2} \sum_{k=0}^{\infty} \frac{\cos((2k+1)\pi x)}{(2k+1)^2}$$

$$-\frac{8}{\pi^3} \sum_{k=0}^{\infty} \frac{1}{(2k+1)^3} \operatorname{sen}((2k+1)\pi x)$$




$$x = 0 \Rightarrow \sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$$

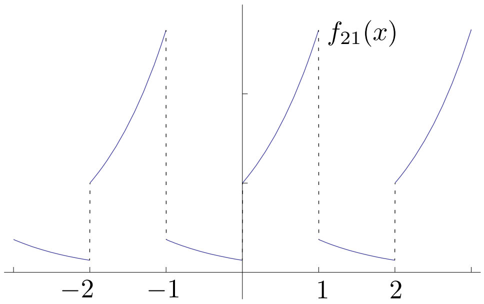
$$x = \frac{1}{2} \Rightarrow \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)^3} = \frac{\pi^3}{32}$$

$$f_{21}(x) = \begin{cases} e^x & 0 < x < 1 \\ e^{-x} & 1 < x < 2 \end{cases}$$

$$f_{21}(x) = (1 - e^{-1}) \cosh 1$$

$$2 \sum_{n=1}^{\infty} \frac{-e^{-1} + (-1)^n}{1 + n^2\pi^2} (\cosh 1 \, \cos(\pi n x) -$$

$$n\pi \operatorname{senh} 1 \, \operatorname{sen}(\pi n x))$$




$$x = 0 \Rightarrow \sum_{n=1}^{\infty} \frac{e^{-1} - (-1)^n}{1 + n^2\pi^2} \cosh 1 = \frac{1}{2} - e^{-1}$$